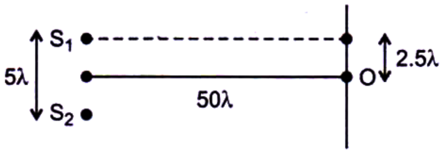
WEEKLY TEST OYJ TEST - 20 R & B
SOLUTION Date 01-09-2019

[PHYSICS]

1

We know that $I_{max} = I_0$



Given that $d = 5\lambda$, Hence $2.5\,\lambda = \frac{d}{2}$

Path diff. $= \frac{dy_n}{D} = \frac{d \times \frac{d}{2}}{10d} = \frac{d}{20} = \frac{\lambda}{4}$

Phase diff. = 90°

$$I = I_0 \cos^2 \frac{\phi}{2} = \frac{I_0}{2}$$

2.

If new value of distance of screen from double slit be D', then

$$\beta' = \frac{\lambda D'}{d'} = \frac{\lambda D'}{(2d)} = \frac{\lambda D}{d} = \beta$$

or $D' = 2D$.

3.

For first minima,

$a \sin\theta_1 = \lambda$

∴ $a = \frac{\lambda}{\sin\theta_1} = \frac{6200 \times 10^{-10}}{\sin 30°} = 1.24 \times 10^{-6}\,\text{m}$

$= 1.24\,\mu\text{m}$.

4.

Suppose n_1th bright fringe of wavelength λ_1 coincides with n_2th bright fringe of wavelength λ_2. Then,

$$\frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d}$$

or $$n_1\lambda_1 = n_2\lambda_2$$

or $$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{10{,}000}{12{,}000} = \frac{5}{6}$$

Let x be the given distance.

$$\therefore \quad x = \frac{n_1\lambda_1 D}{d}$$

Given that $n_1 = 5, D = 2\text{ m},$

$d = 2\text{ mm} = 2\times 10^{-3}\text{m},$

$\lambda_1 = 12000\text{Å} = 12000\times 10^{-10}\text{m} = 12\times 10^{-7}\text{m}$

$$\therefore \quad x = \frac{5\times 12\times 10^{-7}\times 2}{2\times 10^{-3}}$$

$$= 6\times 10^{-3}\text{m} = 6\text{ mm}$$

Hence, correct answer is (a).

5.

Fringe width, $\beta = \frac{\lambda D}{d}$

where λ is the wavelength of light, D is distance between slits and the screen, d is distance between the two slits.

As the D, d remain the same

$$\beta \propto \lambda$$

or $$\frac{\beta'}{\beta} = \frac{\lambda'}{\lambda}$$

or $$\beta' = \frac{\lambda'\beta}{\lambda}$$

Substituting the given values, we get;

$$\beta' = \frac{4000\text{ Å}\times 3\text{ mm}}{6000\text{ Å}}$$

$$= 2\text{ mm.}$$

6.

Fringe width, $\beta = \frac{\lambda D}{d}$

$$\therefore \quad D = \frac{\beta d}{\lambda} = \frac{4\times 10^{-3}\times 0.1\times 10^{-3}}{4\times 10^{-7}} = 1\text{ m.}$$

7.

$$\frac{I_{\text{max.}}}{I_{\text{min.}}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2} = \frac{(2+1)^2}{(2-1)^2} = \frac{9}{1}.$$

8.

For maximum intensity on the screen,

$$d \sin\theta = n\lambda$$

or $$\sin\theta = \frac{n\lambda}{d} = \frac{(n)(2000)}{(7000)} = \frac{n}{3.5}$$

Since, $\sin\theta \not> 1$

$\therefore$ $n = 0, 1, 2, 3$ only.

Thus, only seven maximas can be obtained on both sides of the screen.

9.

$$I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$$

$$I_0 = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$$

or $$\cos\left(\frac{\phi}{2}\right) = \frac{1}{2}$$

or $$\frac{\phi}{2} = \frac{\pi}{3}$$

or $$\phi = \frac{2\pi}{3} = \frac{2\pi}{\lambda}\cdot\Delta x$$

or $$\frac{1}{3} = \frac{1}{\lambda}\cdot\left(\frac{yd}{D}\right)$$

$$\therefore\ y = \frac{\lambda}{3\left(\frac{d}{D}\right)} = \frac{6\times10^{-7}}{3\times10^{-4}} = 2\times10^{-3}\text{ m} = 2\text{ mm}.$$

10.

$$n_1\lambda_1 = n_2\lambda_2$$

$$n\,(7.8\times10^{-5}) = (n+1)(5.2\times10^{-5})$$

or $$n\,(2.6\times10^{-5}) = 5.2\times10^{-5}$$

$\therefore$ $n = 2$.

11.

$$I = 2I_0(1+\cos\delta)$$

When path difference = λ, then phase difference

$$\delta = \frac{2\pi}{\lambda}\times\text{path diff.} = 2\pi$$

$$\therefore\ I_1 = 2I_0(1+\cos 2\pi) = 4I_0 = K \quad ...(i)$$

When path difference = $\lambda/4$, then phase difference

$$\delta = \frac{2\pi}{\lambda}\times\frac{\lambda}{4} = \frac{\pi}{2}$$

$$\therefore\ I_2 = 2I_0\left(1+\cos\frac{\pi}{2}\right) = 2I_0 = \frac{K}{2}.$$

12.

13.

$$7\beta_1 = d_1 = 7\frac{\lambda_1 D}{d} \quad \text{and} \quad 7\beta_2 = d_2 = 7\frac{\lambda_2 D}{d}$$

$$\therefore \quad \frac{d_1}{d_2} = \frac{\lambda_1}{\lambda_2}.$$

14.

Given, $\beta = 1\text{ mm} = (D\lambda/d)$

Distance of 1st bright fringe from the centre,

$$x_n = 2n\left(\frac{D\lambda}{2d}\right)$$

For first bright fringe, $n = 1$

So, $$x_1 = 2\left(\frac{D\lambda}{2d}\right) = \frac{D\lambda}{d} = 1\text{ mm}.$$

15.

$\lambda = 5000\text{ Å}$, $d = 0.2$ mm and $D = 200$ cm

$$x_n = 2n\left(\frac{D\lambda}{2d}\right)$$

$$\therefore \quad x_3 = 2 \times 3\left(\frac{D\lambda}{2d}\right)$$

$$= \frac{3 \times 200 \times 5000 \times 10^{-8}}{0.2 \times 10^{-1}}\text{ cm} = 1.5\text{ cm}.$$

[CHEMISTRY]

24. **(a) :** For the same aryl group, *b.p.* increases as the size of the halogen increases. Thus, C_6H_5I has the highest *b.p.*

25. **(c) :** Boiling point decreases with branching. Therefore, option (*c*) is wrong.

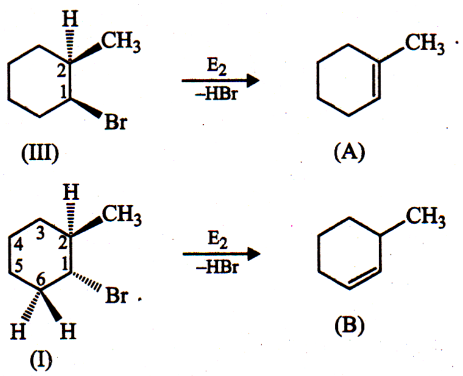
26. **(d) :** Being strained cyclopropane ring readily opens up to form only *n*-propyl bromide. In contrast, reaction (*a*) gives a mixture of *n*-propyl and isopropyl bromides, reaction (*b*) gives isopropyl bromide while reaction (*c*) does not occur at all.

27. **(d) :** As the size of the alkyl group increases, the S_N2 reactivity decreases. Further, C–Cl bond is stronger and more difficult to cleave than C–Br bond. Thus, option (*d*) is correct.

28. **(d) :** 3-Methyl-3-bromohexane is a 3° alkyl halide and hence undergoes solvolysis (nucleophilic substitution reaction with the solvent) by S_N1 mechanism. Since S_N1 reactions do not involve inversion, therefore, option (*d*) is *incorrect* while all other options are correct.

29. (*b*) : During the reaction between the optically active alcohol and *p*-toluenesulphonic acid, the C–O bond to the chiral centre is not broken. Instead O–H bond is broken. As a result the configuration of the alcohol is retained in the tosylate (A). However, when tosylate (A) is

30. (*b*) : In E_2 reactions, *trans*-elimination occurs. Since in compound (III), there is a *trans*-H-atom on C_2 carbon carrying the CH_3 group, therefore elimination occurs readily to give stable alkene (A).



In compound (I), *trans*-H is not available on C_2 but there is a *trans*-H available on C_6, therefore, elimination occurs on the other side to give less stable alkene (B). Compound (II), however, does not have a *trans*-H on either side (*i.e.*, C_2 or C_6), therefore, E_2 reaction does not occur. Thus, option (*b*) is correct.

[MATHEMATICS]

31.

Differentiating both the sides and using Property 17, we have

$xf(x) = \cos x - \cos x + x \sin x - x$, so $f(x) = \sin x - 1$.

Hence $f(\pi/6) = \sin \pi/6 - 1 = -1/2$.

32.

Using Property 17, we have $f'(x) = \sqrt{2 - x^2}$ and thus the given equation reduces to $x^2 - \sqrt{2 - x^2} = 0 \Rightarrow (x^2 + 2)(x^2 - 1) = 0$. Thus the real roots are given by $x = \pm 1$.

33.

$F(x^2) = \int_0^{x^2} f(t)\,dt$, therefore, $x^2(1 + x) = \int_0^{x^2} f(t)\,dt$. Differentiating both sides w.r.t. x using Property 17, we have

$$2x + 3x^2 = f(x^2) \cdot 2x \Rightarrow f(x^2) = 1 + (3/2)x$$

Putting $x = 2$, we have $f(4) = 1 + 3 = 4$.

34.

Putting $t = 1/u$ in I_2 we have

$$I_2 = -\int_e^{\tan x} \frac{u\,du}{1+u^2} = -\int_{1/e}^{\tan x} \frac{u\,du}{1+u^2} + \int_{1/e}^{e} \frac{u\,du}{1+u^2}$$

$$= -I_1 + \frac{1}{2}\int_{1/e}^{e} \frac{2u\,du}{1+u^2}$$

So $$I_1 + I_2 = \frac{1}{2}\log(u^2+1)]_{1/e}^{e} = \frac{1}{2}\left[\log\left(e^2+1\right) - \log\left(\frac{e^2+1}{e^2}\right)\right]$$

$$= \frac{1}{2}\times 2 = 1.$$

35.

Let $f(x) = \dfrac{x^7 - 3x^5 + 7x^3 - x}{\cos^2 x}$ then $f(-x) = -f(x)$ so $\int_{-1}^{1} f(x)\,dx$

$= 0$ (Property 11)

Let $g(x) = \cos^{-1} x$ then $I_1 = \int_{-1}^{1} \cos^{-1} x\,dx = \int_{-1}^{1} \cos^{-1}(-x)\,dx$ (Property 8)

$$= \int_{-1}^{1} (\pi - \cos^{-1} x)\,dx = 2\pi - I_1$$

Thus $I_1 = \pi$. Hence $I = \pi$.

36.

$$\int_0^n [x]\,dx = \sum_{i=1}^{n}\int_{i-1}^{i} [x]\,dx = \sum_{i=1}^{n}\int_{i-1}^{i} (i-1)\,dx$$

$$= \sum_{i=1}^{n} (i-1) = \frac{n(n-1)}{2}.$$

37.

$$\text{Required limit} = \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{2n} \frac{r/n}{\sqrt{1+r^2/n^2}}$$

$$= \int_0^2 \frac{x}{\sqrt{1+x^2}}\,dx = \sqrt{1+x^2}\Big|_0^2 = \sqrt{5} - 1.$$

38.

Since the period of $|\sin x| + |\cos x|$ is $\pi/2$ so

$$\int_0^{n\pi+t} (|\sin x| + |\cos x|)\,dx = 2n\int_0^{\pi/2} (|\sin x| + |\cos x|)dx + \int_0^{t} (|\sin x| + |\cos x|)\,dx$$

$$= 2n\int_0^{\pi/2} (\sin x + \cos x)\,dx + \int_0^{t} (\sin x + \cos x)\,dx$$

$$= (2n)(2) + \sin t - \cos t + 1 = (4n+1) + \sin t - \cos t.$$

39.

$$I=\int_0^{\pi/2}\frac{dx}{1+\tan^3 x}=\int_0^{\pi/2}\frac{dx}{1+\tan^3(\pi/2-x)}$$

$$=\int_0^{\pi/2}\frac{dx}{1+\cot^3 x}$$

$$=\int_0^{\pi/2}\frac{\tan^3 x}{1+\tan^3 x}dx=\int_0^{\pi/2}\left(1-\frac{1}{1+\tan^3 x}\right)dx=\frac{\pi}{2}-I.$$

Thus $I=\pi/4$.

40.

$$\int_0^1 x(1-x)^{99}\,dx=\int_0^1(1-x)(1-(1-x))^{99}\,dx \qquad \text{(Property 8)}$$

$$=\int_0^1(x^{99}-x^{100})\,dx=\left.\frac{x^{100}}{100}-\frac{x^{101}}{101}\right|_0^1=\frac{1}{10100}.$$

41.

Since the period of the function $x-[x]$ is 1 so

$$\sum_{n=1}^{1000}\int_{n-1}^{n} e^{x-[x]}\,dx=\int_0^{1000} e^{x-[x]}\,dx$$

$$=1000\int_0^1 e^{x-[x]}\,dx=1000\int_0^1 e^x\,dx=1000\,(e-1).$$

42.

$$f'(x)=\cos\sqrt{x^2}\,\frac{d}{dx}(x^2)-\cos\sqrt{1/x^2}\,\frac{d}{dx}(1/x^2)$$

(Using Property 17)

$=2x\cos x+\dfrac{2}{x^3}\cos(1/x)$. Hence $f'(1)=4\cos 1$.

43.

Differentiating the given function, we get

$$F'(x)=t+1]_{t=x}\frac{dx}{dx}-[t+1]_{t=0}\frac{d0}{dx}=x+1.$$

This is positive for all $x\in[2,3]$, so F is an increasing function in this interval. Therefore its greatest value is $F(3)=\int_0^3(t+1)\,dt$ and its least value is $F(2)=\int_0^2(t+1)dt$, so that the required difference between these values is $\int_0^3(t+1)dt-\int_0^2(t+1)dt=\int_2^3(t+1)dt=\dfrac{7}{2}$.

44.

$\frac{dy}{dx} = (x-1)(x-2)^2$ so $\frac{d^2y}{dx^2} = (x-2)(3x-4)$. The points of inflection are given by $\frac{d^2y}{dx^2} = 0$ so $x = 2$, $x = 4/3$ are points of inflection.

45.

$$I = \int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos x} = \int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos(\pi - x)} \qquad \text{(Property 8)}$$

$$= \int_{\pi/4}^{3\pi/4} \frac{dx}{1-\cos x}$$

$$2I = \int_{\pi/4}^{3\pi/4} \left(\frac{1}{1+\cos x} + \frac{1}{1-\cos x}\right) dx = \int_{\pi/4}^{3\pi/4} \frac{2}{1-\cos^2 x} dx$$

$$= -2 \cot x \Big|_{\pi/4}^{3\pi/4} = 4. \text{ Hence } I = 2$$